NAME: _____

Math 324 FALL 2004: Section 3 MWF 12-1
Final Exam Date: Dec 15, 2004

Instructions: Answer all questions. Show working where it is useful to do so. You have 50 minutes. To allow others to fully concentrate at the end please do not leave in the last 10 minutes. You should submit your pages of notes with your test paper.

Question 1. (*50 points*)

Suppose that two random variables X and Y have the following joint density function:

$$f(x,y) = \begin{cases} k\left(x+y^2\right) & 0 \le x \le 2, 0 \le y \le 2 \\ 0 & \text{Otherwise} \end{cases}$$

where k is a constant.

(a) Find the value of k that makes this a valid density function.

(b) What is the probability that both X and Y are above 1?

(c) What is the probability that the difference between X and Y is at most 1?

(d) Determine the marginal distributions of X and Y.

(e) Are X and Y independent?

(f) What is the conditional pdf of Y given $X = x$?

(g) Suppose that $X = 1$. What is the probability that $Y < 1$?

Question 2. (*20 points*)

A quality inspector at a soda factory takes a random sample of 50 bottles produced each hour and measures the volume of soda in each bottle. The soda company does not want to fill bottles over the quantity stated because it will cost more money and they will face serious fines from the federal trade commission if they significantly under-fill bottles. If the filling machine is working perfectly then on average each bottle is filled with 600 ml of soda with standard deviation 5 ml. The inspector decides that he will only have the filling machine adjusted if the mean volume in each bottle is below 599ml or above 601ml.

(a) Assuming the machine is working correctly what is the chance for a given hour that he will recommend it be changed.

(b) Is his strategy a sound one? Why or why not?

Question 3. (*30 points*)

Suppose that you have a random sample $X_1, \ldots, X_n$ from a distribution with the following pdf:

$$f(x;\theta) = \theta x^{\theta-1}$$

where $0 < x < 1$ and $0 < \theta < \infty$.

(a) Find the Method of Moments Estimator for θ.

(b) Find the Maximum Likelihood Estimator for θ.

Question 4. (*25 points*)

A machine produces bolts in a factory. Acceptable bolts have a mean diameter 0.25in. A quality inspector working for the factory takes a random sample of 50 bolts from all the bolts produced each hour and decides whether or not the machine is producing acceptable bolts. If the machine is not producing acceptable bolts it needs to be adjusted. For one particular hour the mean diameter of the 50 bolts was 0.27in with standard deviation 0.05 inches.

(a) Give a 95% confidence interval for the mean diameter of the bolts produced.

(b) Explain how you could carry out a hypothesis test using the confidence interval. What would you conclude if you did carry out this test?

Question 5. (*50 points*)

A biologist is doing research on how a particular growth hormone affects weight gain in mice. She takes 39 mice each 7 days old and randomly chooses 17 of these mice to receive the growth hormone

and the remaining 22 mice to not receive any growth hormone (these are the control group mice). Both groups of mice are fed the same diet. After 14 days she weighs each of the mice and records their weights. The group of mice that received the growth hormone had mean weight gain of 26.3g and standard deviation 3.3g. The control group had mean weight gain of 24.4g and standard deviation 2.8g.

For this question let μ_1 be the mean weight gain of any mouse who receives the growth hormone and let μ_2 be the mean weight gain of any mouse who receives the same treatment as the control group.

(a) State appropriate null and alternative hypotheses for testing whether or not there is a difference in mean weight gain.

(b) Calculate the test statistic value (you should not assume pooling).

(c) What are the appropriate degrees of freedom for this test statistic?

(d) Suppose that you are testing at the 1% level of significance. What conclusion would you draw? Be sure to explain this in terms of the original situation.

(e) Give a 99% confidence interval for the difference in mean weight gain.

(f) What assumptions are you making for the confidence interval and hypothesis test to be valid.

Question 6. (*25 points*)

An observational medical study was carried out to examine the association between high blood pressure and increased risk of death from cardiovascular disease. There were 2676 men with low blood pressure and 3338 men with high blood pressure. In the low-blood-pressure group, 21 men died from cardiovascular disease. In the high blood pressure group 55 died.

(a) Give a 95% confidence interval for the difference in proportions.

(b) Carry out an appropriate hypothesis test to determine whether the death rate is higher among men with higher blood pressure. Be sure to clearly state your null and alternative hypothesis. You should state in words what you conclude from your hypothesis test.