324 Lecture #? Monday 13th Sept (1)

For an experiment with equally likely outcomes the probability of an event A is

$$P(A) = \frac{N(A)}{N}$$

If N is small then it is easy to count both N and $N(A)$

However often N is very large and we need some ~~techniques~~ methods for counting in a more systematic way. That is the topic for todays lecture

ordered pairs — order matters

eg two coin tosses
~~H~~HT is different from TH

~~An ordered pair~~

If the first element or object of an ordered pair can be selected in n_1 ways and ~~the second~~ for each of these n_1 ways the second element of the pair can be selected in n_2 ways then the total number of pairs is $n_1 n_2$. ②

eg ice cream store

31 flavors

10 toppins

$31 \times 10 = 310$ different ways to combine icecream and toppings.

A k-tuple generalizes the ordered pair to k elements. Note each element is selected from a completely different set of elements or perhaps the same list with replacement.

<u>Product rule</u> n_i number of possible choices for ith element for each possible choice of the first i-1 elements

$n_1 \times n_2 \times n_3 \times \ldots \times n_k$ possible k-tuples

eg Icecream store

31 flavours
10 toppins
3 serving sizes
2 cup, cone

$31 \times 10 \times 3 \times 2$
$= 1860$ possible ways

Permutations

consider a fix set of n elements for which we want to take anordered sequence of k elements.

eg select 3 elements from

$\{1,2,3,4,5,6\}$

$\{1,2,3\}$ $\{6,4,3\}$
$\{3,2,1\}$

permutations

The number of permutations of size k that can be selected from n objects is written $P_{k,n}$ (sometimes ^nP_k | $_nP_r$)

$$P_{k,n} = n(n-1)(n-2)\ldots(n-k+2)(n-k+1)$$

ways to choose first element

ways to choose second element.

ways to choose the kth element.

Factorial notation

$n! = \cancel{\quad} n(n-1)(n-2)\cdots(2)(1)$

positive integer

note $1! = 1$

$0! = 1$

$$\boxed{P_{k,n} = \frac{n!}{(n-k)!}}$$

Combinations

order is not important.

~~the number of ways to choose~~

a combination is an unordered subset of size k choosen from n distinct objects. Denoted $\binom{n}{k}$ | $C_{k,n}$ | ${}^{n}C_{k}$ | ${}_{n}C_{k}$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Examples

1. In how many ways can 10 people be seated on a bench if only 4 seats are available?

$$_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \times 9 \times 8 \times 7 = 5040$$

2. How many 4-digit numbers can be formed if

(a) repetitions are allowed

$$9 \times 10 \times 10 \times 10 = 9000$$

0 not allow

(b) not allowed

$$9 \times 9 \times 8 \times 7 = 4536$$

not allow 0

(c) last digit must be 0 and no repetitions

$$9 \times 8 \times 7 = 504$$

3. 4 different Math books, 6 different physics books, 2 chem books. How many different arrangements if

(a) the books on each subject must be together

$$\underbrace{4!}_{\text{math}}\,\underbrace{6!}_{\text{physics}}\,\underbrace{2!}_{\text{chem}}\,\underbrace{3!}_{\text{three groups}} = 207{,}360$$

(b) the math book only together

treat math books as single book

$$\underbrace{9!}_{\text{all book groups}}\,\underbrace{4!}_{\text{math books}} = 8{,}709{,}120$$

4. How many ways can a committee of 5 people be choosen from 9 people

$$\binom{9}{5} = {}^{9}C_{5} = \frac{9!}{(9-5)!\,5!} = \frac{9\times8\times7\times6}{4!} = 126$$

5. Out of 5 Mathematicians and 7 physicists, a committee of 2 math, 3 phys is to be formed. How many ways if

(a) any mathematician and any physicist

$$ {}^{5}C_2 \times {}^{7}C_3 = \frac{5!}{3!2!}\,\frac{7!}{4!3!} = 10 \times 35 = 350 $$

math phys

(b) One particular physicists must be on committee

$$ {}^{5}C_2 \, {}^{6}C_2 = \frac{5!}{3!2!}\,\frac{6!}{4!2!} = 10 \times 15 = 150 $$

(c) two mathematicians cannot be on the committee

$$ {}^{3}C_2 \, {}^{7}C_3 = 3 * 35 = 105 $$