Lecture 14

A <u>binomial experiment</u> is an experiment which

1. Consists of n smaller experiments called <u>trials</u>, where n is fixed before the experiment is carried out.
2. Each trial consists of only two possible outcomes (which we denote by S and F)
   (S ↑ success, F ↑ failure)
3. The trials are independent
4. The probability of success on any trial is constant (denote this by p)

eg Tossing a coin 10 times H-S, T-F

Sampling with replacement from a set of widgets Good-S, Faulty-F

A binomial random variable X is defined as

$X =$ "the number of successes in the n trials"

Note that a binomial random variable is equivalent to taking the sum of n independent Bernoulli random variables

Notation

"is distributed"

$X \sim Bin(n, p)$

X: random variable; Bin: distribution; n, p: parameters

Binomial pdf

$$b(x; n, p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, 2, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

$b(x; n, p)$: probability of x successes

$\binom{n}{x}$: # sequences of x successes

p^x: probability of x successes

$(1-p)^{n-x}$: probability of $n-x$ successes

If $X \sim Bin(n,p)$ then

$E[X] = np$ $\quad Var[X] = np(1-p)$

Some examples

1. Consider a box containing 10 black and 20 white marbles. Let $X =$ "number of black marbles selected". Assume that we sample with replacement 5 marbles.

what's probability of 2 black marbles?

$$P(X=2) = \binom{5}{2}\left(\frac{10}{30}\right)^2\left(\frac{20}{30}\right)^3 \qquad X \sim Bin\left(5, \frac{10}{30}\right)$$

what is probability of at least 1 black marble

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \binom{5}{0}\left(\frac{10}{30}\right)^0\left(\frac{20}{30}\right)^5$$

what is the mean number of black marbles

$$E[X] = np = 5\left(\frac{1}{3}\right) = \frac{5}{3} = 1\frac{2}{3}$$

What is the variance of the number of black marbles sampled?

$$Var(X) = np(1-p) = 5\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{10}{9}$$

2. Suppose that 20% of the bolts produced by a machine are defective. Suppose that 4 bolts are examined.

What is the probability that less than 2 bolts are defective?

Let X = "number of defective bolts"

$X \sim Bin(4, 0.2)$

$$P(X<2) = P(X=0) + P(X=1)$$

$$= \binom{4}{0} 0.2^0 0.8^4 + \binom{4}{1} (0.2)^1 (0.8)^3$$

$$=$$

3. What is the probability of getting a sum of 7 in at least 2 of 3 rolls of a pair of dice?

Let X = "number of rolls with sum of 7"

$X \sim Bin(3, \frac{1}{6})$

$$P(X \geq 2) = P(X=2) + P(X=3)$$
$$= \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{3-2} + \binom{3}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{3-3}$$

4. Suppose the probability of finding a defective bolt is 0.1. What is the mean and std dev of the number of defective bolts in a batch of 400 bolts?

Let X = "number of defective bolts"

$X \sim Bin(400, 0.1)$

$E[X] = 400(0.1) = 40$

$SD(X) = \sqrt{Var(X)} = \sqrt{400(0.1)(0.9)} = \sqrt{36} = 6$

5. Suppose that only 25% of all drivers come to a complete stop at an intersection having flashing red lights when no other cars are visible. What is the probability, that of 10 randomly chosen drivers coming to such an intersection

a) at most 6 come to a complete stop
b) exactly 6 come to a complete stop
c) at least 6 come to a complete stop
d) How many of the next 10 drivers could be expected to stop?

a) Let X = "number who stop"

$X \sim Bin(10, .25)$

$$P(X \leq 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$$

$$= \binom{10}{0}(.25)^0(.75)^{10} + \binom{10}{1}(.25)^1(.75)^9 + \binom{10}{2}(.25)^2(.75)^8 + \binom{10}{3}(.25)^3(.75)^7 + \binom{10}{4}(.25)^4(.75)^6 + \binom{10}{5}(.25)^5(.75)^5 + \binom{10}{6}(.25)^6(.75)^4$$

$= .0563 + .1877 + .2816 + .2502 + .1460$
$+ .0584 + .01622 = 0.9965$

(b) $P(x=6) = \binom{10}{6} .25^{6} .75^{10-6}$

$= .01622$

(c) $P(x \geq 6) = 1 - P(x \leq 6) + P(x=6)$

$= 1 - .9965 + .01622$

$= .0147$