Lecture 31

To summarize our discussion thus far use this table

Confidence Intervals for μ at level of confidence $100(1-\alpha)\%$

Population Distribution	sample size	Sigma known	CI formula	Approximate
Normal	any	yes	$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$	no
Any	large	yes	$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$	Yes by CLT
Normal	any	no	$\bar{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$	no
Any	large	no	$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$	Yes by CLT
Any	moderate	no	$\bar{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$	Yes by CLT

One sided CI are same as above only replace $z_{\alpha/2}$ or $t_{n-1,\alpha/2}$ with z_α or $t_{n-1,\alpha}$ and only look at either $+$ (for upper) or $-$ (for lower) only.

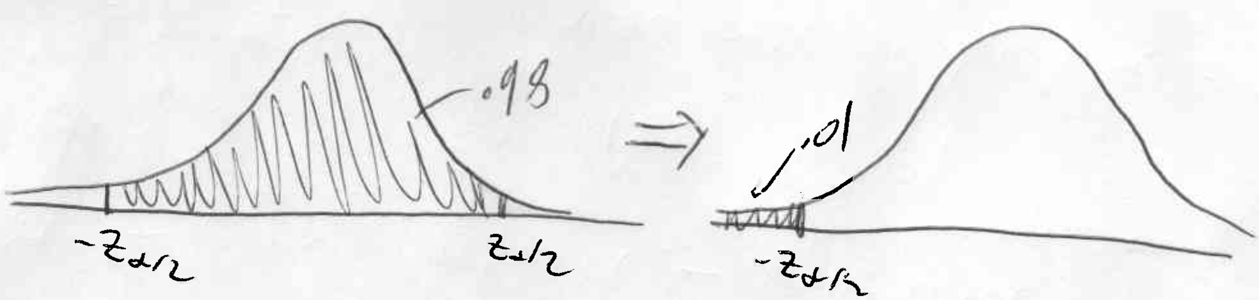
Some examples

1. Suppose we are interested in assessing the accuracy of a laboratory scale. To do this a standard weight is used (known to weigh 10g). The scale readings are normal distributed with unknown mean. The sd of scale readings is known to be .0002g.

   (a) what is a 98% CI for μ? Assume that the weight is weighed 5 times with mean weight 10.00023.

   (b) How many measurements must be averaged to get $\pm .0001$ with 98% confidence

(a) First find $z_{\alpha/2}$ for $\alpha = .02$



from table $\Phi(z) = .01$ closest is -2.33

So 98% CI for μ is

$$10.00023 \pm 2.33 \frac{(.0002)}{\sqrt{5}}$$

$$\Rightarrow 10.00023 \pm .00021$$

$$(10.00002, 10.00044)$$

Note this probably means the scale is slightly biased

(b) recall

$$n = \left(2 z_{\alpha/2} \frac{\sigma}{w}\right)^2$$

$$w = 2(.0001) = .0002$$

$$n = \left(2(2.33)\frac{(.0002)}{(.0002)}\right)^2$$

$$= (2(2.33))^2 = 21.7156$$

So take at least 22 measurements

Example 2

A radio talk show invites its listener to enter a dispute about a proposed new ~~tax~~ system. In all 958 people call and suggest a mean tax of \$9740 per year. The standard deviation of the responses is \$1125.

(a) If the station computes a 95% CI for μ (the tax that all citizens would like to pay) what will it be.

(b) Is this result trustworthy? why?

(a) Since $n = 958$ is so large, no matter what the distribution of the underlying ~~mean~~ tax values we can use

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \text{for } 95\% \text{ CI}$$

$$z_{\alpha/2} = 1.96$$

$$9740 \pm 1.96 \frac{(1125)}{\sqrt{958}}$$

$$9740 \pm 71.24$$

So 95% CI for mean tax is

$$(9668.76, 9811.24)$$

(b) the result is not trustworthy because the sample is not random.

Example 3

The Level of various substances in the blood of Kidney dialysis patients is of concern because kidney failure and dialysis lead to serious problems. A researcher measured the levels of phosphate in a particular patients blood for six separate visits to a particular clinic. Note that phosphate levels

for an individual tend to vary normally over time.
In mg/dl the readings were

5.6, 5.1, 4.6, 4.8, 5.7, 6.4

(a) What is a 90% CI for the mean phosphate level in this patients blood?

First find $\bar{X} = \dfrac{5.6 + 5.1 + \cdots + 6.4}{6} = 5.3666$

what is s? $\quad \Sigma x^2 = 175.02$

$$s = \sqrt{\frac{175.02 - 6(5.3666)^2}{6-1}} = .6653$$

So 90% CI is given by

$$\bar{x} \pm t_{6-1,\alpha/2} \frac{s}{\sqrt{n}}$$

$$t_{5,.05} = 2.015$$

So

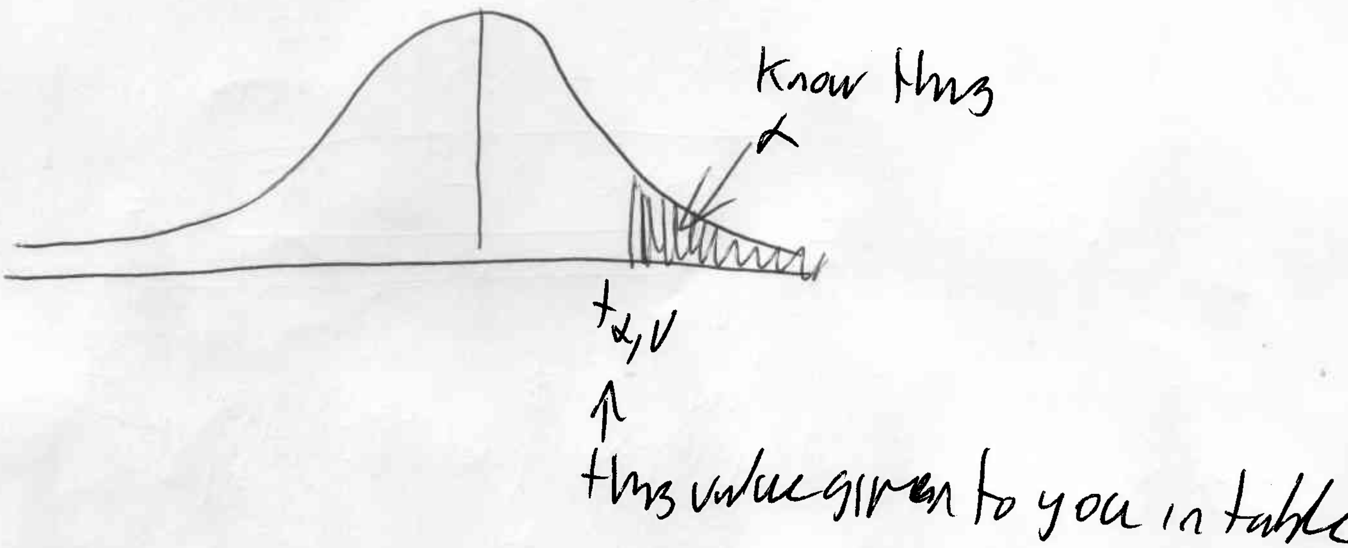
$$5.3666 \pm 2.015 \frac{(.6653)}{\sqrt{6}}$$

ie $5.3666 \pm .5473$

so 90% CI for μ is

$(4.8193, 5.9139)$

<u>Note</u> the t-distribution table gives areas of the following form



So $t_{.05,5}$ was given by looking in $\alpha = .05$ column and going to $\nu = 5$ row.

Confidence intervals for σ^2 (of normal population)

It can be shown that for $X_1, \dots, X_n$ a random sample from $N(\mu, \sigma^2)$. The rv

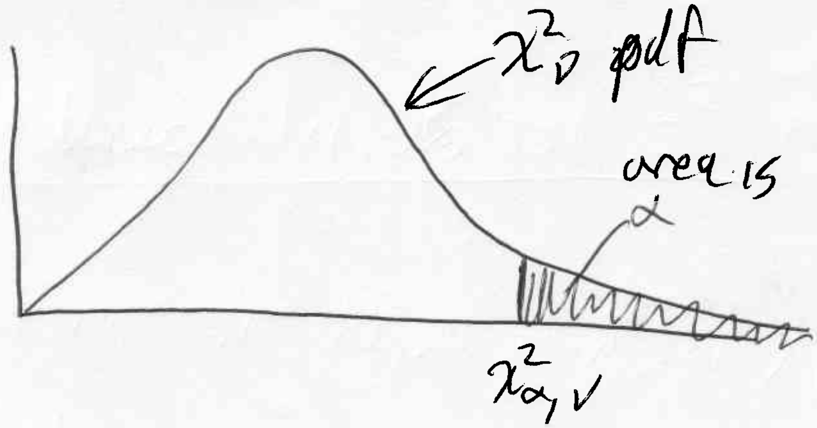
$$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum (X_i - \bar{X})^2}{\sigma^2}$$

has χ^2 (chi-square) distribution with n-1 df

(recall that the chi-square is a special case of the gamma distribution. Based on this knowledge it can be shown that

$$\left((n-1)S^2 / \chi^2_{\alpha/2, n-1} \; , \; \frac{(n-1)S^2}{\chi^2_{1-\alpha/2, n-1}} \right)$$

is a $100(1-\alpha)\%$ CI for σ^2. Note that $\chi^2_{\alpha, \nu}$ is from χ^2 dist



Example

Suppose the amount of lateral expansion was determined for 9 pulse powered gas metal arc welds used in natural gas tanks. The resulting sample std dev was 2.81. What is a 95% CI for σ^2? Assume normality.

A 95% CI for σ^2 is

$$\left(\frac{(9-1)\,2.81^2}{2.18},\ \frac{(9-1)\,2.81^2}{17.534} \right)$$

ie 95% CI for σ^2 is given by
$(3.60, 28.98)$