Lecture 33

Today we see hypothesis tests for μ and p (the population proportion)

Normal population with σ known

Recall that if

$$X_1, \ldots, X_n$$

are from $N(\mu, \sigma^2)$ the $\bar{X}$ has normal distribution $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$. If $H_0: \mu = \mu_0$ is true where μ_0 is some constant then.

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \quad \text{has } N(0,1) \text{ distribution.}$$

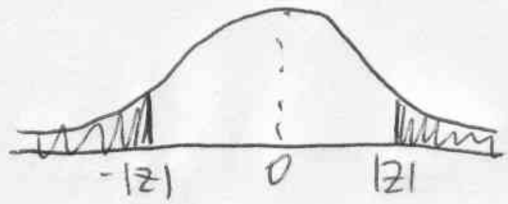
This leads to the following framework for testing hypotheses about μ.

1) State Null/Alternative
2) Compute the value of z (sample version of Z)
3) Find a P-value (or rejection region) using $N(0,1)$ dist.

null $\qquad\qquad\qquad\qquad$ Alternative

$H_0: \mu = \mu_0$ vs $H_A: \mu \neq \mu_0$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$



P value $= 2P(Z > |z|)$

or rejection region is

$$z \geq z_{\alpha/2} \quad \text{or} \quad z \leq -z_{\alpha/2}$$

where $z_{\alpha/2}$ is chosen same as in CI context.

Large Sample

Recall that no matter the population distribution if the sample size is large the CLT tells us $\bar{X}$ is approx $N(\mu, \frac{\sigma}{\sqrt{n}})$ and furthermore s is likely to be close to σ.

Again assume that H_0 is true so that $\mu = \mu_0$

so

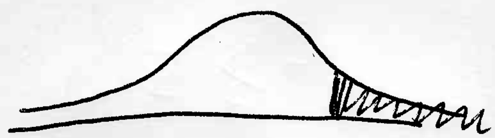
$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$ has approx $N(0,1)$ distribution.

So our previous discussion holds true. ie

$H_0: \mu \leq \mu_0 \qquad H_A: \mu > \mu_0$

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and Pvalues/rejection regions from $N(0,1)$ dist.

Pvalue $= P(Z > z)$ 

rejection region is $z > z_\alpha$

Note how P-value changed. (this is due to the different alternative).

<u>Normal population σ unkown</u>

Recall from CI discussion and assuming $\mu = \mu_0$ that

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$ has t distribution with $n-1$ degrees of freedom.

In a similar manner to before

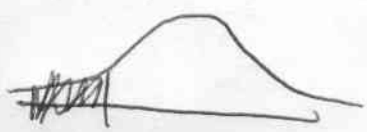
$H_0: \mu \geq \mu_0 \quad H_A: \mu < \mu_0$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

use the t-distribution to find rejection region or p-value.

in this case rejection region is $t \leq -t_{\alpha, n-1}$

and pvalue $= P(T < t)$



Hypothesis tests for proportions

Recall that if n is large $\hat{p}$ is approximately $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$

If we assume that $p = p_0$ then

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

should have approximately $N(0,1)$ distribution.

this means we can test hypotheses of the form

$H_0: p = p_0$

vs

$H_A: p \neq p_0$ (or $H_A: p > p_0$ or $H_A: p < p_0$)

using

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

and $N(0,1)$ distribution for p-values (or rejection regions).