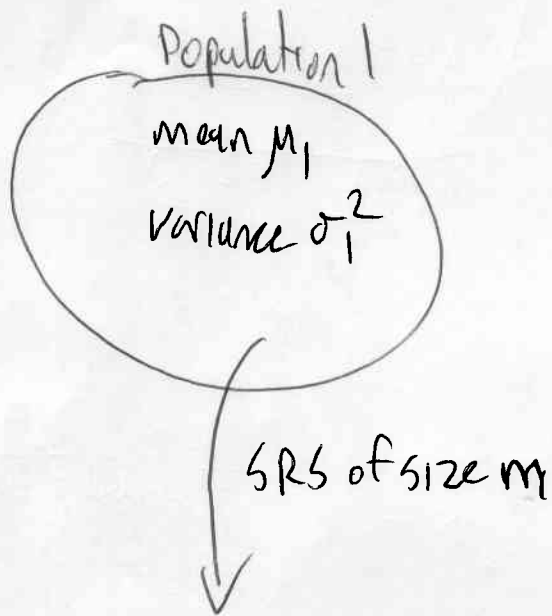


Lecture 35

①

Today we begin the topic of inference for two samples. As with one sample problems we will use confidence intervals and hypothesis tests as our primary tool.

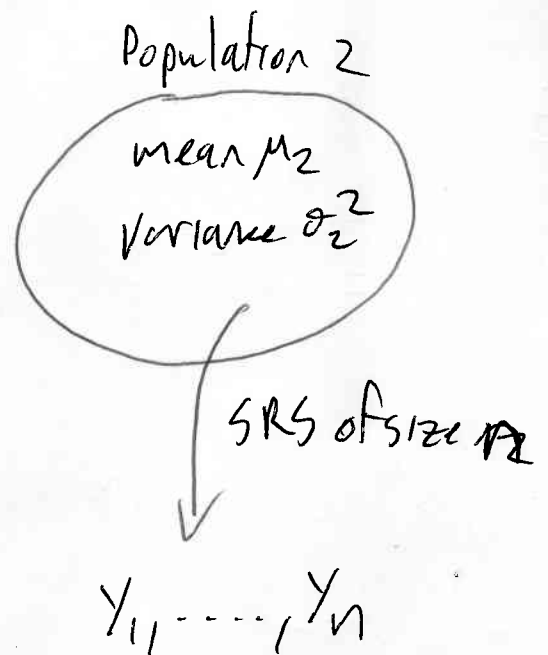
Basic situation



$$x_1, \dots, x_m$$
$$\text{sample mean } \bar{x} = \frac{\sum x_i}{m}$$

sample standard deviation

$$s_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{m-1}}$$



sample mean $\bar{y} = \frac{\sum y_i}{n}$

$$s_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}}$$

Idea: use sample data $\bar{x}, \bar{y}, s_x, s_y$ to

make inferences about μ_1, μ_2 (or σ_1^2, σ_2^2)
and possible differences between them.

Typically we examine (or want to know about)

$$\mu_1 - \mu_2$$

the difference between the two means. Note
that if μ_1 and μ_2 are equal i.e. $\mu_1 = \mu_2$ then

$$\mu_1 - \mu_2 = 0.$$

Note that

$\bar{X} - \bar{Y}$ is an unbiased estimator of $\mu_1 - \mu_2$
and $E[\bar{X} - \bar{Y}] = \mu_1 - \mu_2$

$$\text{Var}(\bar{X} - \bar{Y}) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\Rightarrow \sigma_{\bar{X} - \bar{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(3)

If we assume that both populations are large then by the CLT \bar{X} and \bar{Y} should have approximately normal distributions. This means $\bar{X} - \bar{Y}$ should also have approximately normal distribution. Therefore

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$$

is a random variable with approx $N(0, 1)$ distribution.

Confidence intervals

Following similar derivations to what has been used in the past it can be shown that a

$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$ is given by

$$\bar{x} - \bar{y} \pm Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_1}}$$

when σ_1^2, σ_2^2 are known. (Note exact when populations are both normal, approximate otherwise)

(4)

If both ~~population~~ sample sizes n_1 and n_2 are large then it is reasonable to use s_1^2 and s_2^2 in place of σ_1^2 and σ_2^2 (if unknown) in the prior formula.

It can be shown that if σ_1^2 and σ_2^2 are at least approximately known and equal sized samples are to be taken then to get a CI of width w take

$$n = n_1 = n_2 = \frac{4z_{\alpha/2}^2 (\sigma_1^2 + \sigma_2^2)}{w^2}$$

Round up to the nearest whole integer.

Hypothesis Testing

Assume that the true difference between μ_1 and μ_2 is Δ_0 (some constant value). Often $\Delta_0 = 0$

This means that

$$Z = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad \text{will have } N(0,1)$$

⑤
distribution if both populations are normally distributed
using these assumptions leads us to ...

See next lecture