

Lecture 36

(1)

Hypothesis Testing for differences between two means

(with $\sigma_1 = \sigma_2$ known)

As discussed last time, the statistic

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma^2}{m} + \frac{\sigma^2}{n}}}$$

has $N(0, 1)$ distribution. For hypothesis testing purposes we typically assume, under the null hypothesis, that the difference ~~between~~ $\mu_1 - \mu_2 = \Delta_0$

where Δ_0 is some constant. Often we are interested in checking whether there is any difference or not so $\Delta_0 = 0$ in many cases.

$$H_0: \mu_1 - \mu_2 = \Delta_0$$

$$H_A: \mu_1 - \mu_2 \neq \Delta_0$$

alternative notation

$$\left(\begin{array}{l} H_0: \mu_1 = \mu_2 + \Delta_0 \\ H_1: \mu_1 \neq \mu_2 + \Delta_0 \end{array} \right)$$

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test statistic

$$z = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$$

P-values are from the standard normal distribution.

As is usual the alternative hypothesis tells which tail it use for the P-value

$$H_A: \mu_1 - \mu_2 \neq 0$$

$$2P(Z > |z|)$$



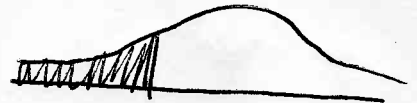
$$H_A: \mu_1 - \mu_2 > 0$$

$$P(Z > z)$$



$$H_A: \mu_1 - \mu_2 < 0$$

$$P(Z < z)$$



Note that when m, n are large and σ_1^2, σ_2^2 are unknown we may safely substitute s_1^2, s_2^2 into above test statistic.

Example 1

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Suppose we have the following data

	n	mean	std dev
Method 1	129	107.6	1.3
Method 2	129	123.6	2.0

where method 1 & 2 are two methods of building a ^{steel} girder and the measurements are breaking strengths.

Find a 95% CI for the difference in means between the two methods. Then using a hypothesis test check whether method 2 has a higher mean than method 1 by more than 15 units.

Let μ_1 = population mean of method 1

μ_2 = population mean of method 2

a 95% CI for $\mu_1 - \mu_2$ is given by

$$\bar{x} - \bar{y} \pm Z_{\alpha/2} \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

$$107.6 - 123.6 \pm 1.96 \sqrt{\frac{1.3^2}{129} + \frac{2.0^2}{129}}$$

$$107.6 - 123.6 \pm 0.41$$

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$$-16 \pm 0.41$$

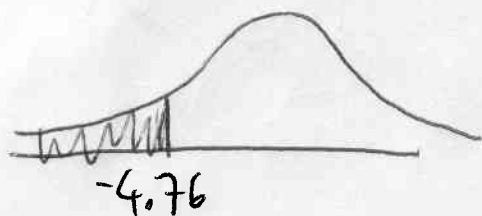
so a 95% CI for $\mu_1 - \mu_2$ is
 $(-16.41, -15.59)$

$$H_0: \mu_1 - \mu_2 \geq -15$$

$$H_A: \mu_1 - \mu_2 < -15$$

$$z = \frac{(107.6 - 123.6) - (-15)}{\sqrt{\frac{1.3^2}{129} + \frac{2.0^2}{129}}} = -4.76$$

$$P\text{-value} = P(Z < -4.76) < P(Z < -3.49) = .0002$$



Since p-value is very small we can reject null hypothesis in favor of alternative. i.e. that μ_2 is more than 15 units above μ_1 .

Example 2

Are male students more likely to be bored than their female counterparts? A study was carried out on 97 randomly chosen male students and 148 female students. Boredom was measured using a scale called the "Boredom Proneness Rating". The resulting data was

<u>Gender</u>	Sample size	mean	sample SD
Male	97	10.40	4.83
Female	148	9.26	4.68

A higher score means more boredom

Find a 99% CI for the difference between means.

Test the hypothesis that male students are more bored than female students.

μ_1 = mean boredom scale Males, μ_2 = mean boredom scale Females.
So a 99% CI for $\mu_1 - \mu_2$ is given by

$$10.40 - 9.26 \pm 2.576 \sqrt{\frac{4.83^2}{97} + \frac{4.68^2}{148}}$$

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$$1.14 \pm 1.60$$

So 99% CI for $\mu_1 - \mu_2$ is

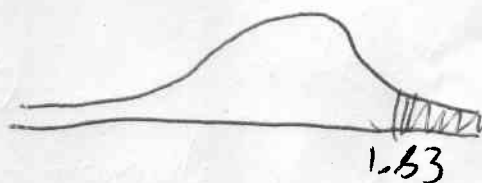
$$(-0.47, 2.75)$$

Hypothesis test

$$H_0: \mu_1 \leq \mu_2 \quad \text{ie } \mu_1 - \mu_2 \leq 0$$

$$H_A: \mu_1 > \mu_2 \quad \mu_1 - \mu_2 > 0$$

$$z = \frac{10.40 - 9.26}{\sqrt{\frac{4.83^2}{97} + \frac{4.68^2}{148}}} = 1.83$$



$$\begin{aligned} P\text{value} &= P(Z > 1.83) = 1 - \Phi(1.83) \\ &= 1 - .9664 \\ &= .0336. \end{aligned}$$

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since the P -value < 0.05
We have weak evidence to show that males are
more easily bored than females