Homework #1 Selected Solutions

42(a) mean of the x_i's $\Rightarrow \bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

median of the x_i's $\Rightarrow x_{[\frac{n}{2}]}$ This is notation for order statistics. It means the $\frac{n}{2}$th observation in an ordered list.

$y_i = x_i + c$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{\sum_{i=1}^{n}(x_i + c)}{n} = \frac{\sum_{i=1}^{n} x_i + nc}{n} = \bar{x} + c$$

median of the y_i's $\quad y_{[\frac{n}{2}]} = x_{[\frac{n}{2}]} + c$

(b) $y_i = cx_i$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{\sum_{i=1}^{n} cx_i}{n} = c\frac{\sum_{i=1}^{n} x_i}{n} = c\bar{x}$$

median of y_i's $\quad y_{[\frac{n}{2}]} = cx_{[\frac{n}{2}]}$

68a $\sum_{i=1}^{n}(x_i-c)^2$ want to find c which minimizes this value

$$\frac{d}{dc}\sum_{i=1}^{n}(x_i-c)^2 = \sum_{i=1}^{n}\frac{d}{dc}(x_i-c)^2$$
$$= \sum_{i=1}^{n}2(x_i-c)(-1)$$

set $=0$

$$-2\sum_{i=1}^{n}(x_i-c)=0$$
$$\Rightarrow \sum_{i=1}^{n}x_i - nc = 0$$
$$\Rightarrow \sum_{i=1}^{n}x_i = nc$$
$$\Rightarrow c = \frac{\sum_{i=1}^{n}x_i}{n} = \bar{x}$$

(b) by part (a) $\sum_{i=1}^{n}(x_i-\bar{x})^2 \leq \sum_{i=1}^{n}(x_i-\mu)^2$

with equality only if $\bar{x}=\mu$

7B (a) $y_i = x_i - \bar{x} \quad i = 1, \ldots, n$

Note
$\left.\begin{matrix} s_x \\ s_y \\ s_z \end{matrix}\right\}$ standard deviations of x, y, z

$$s_x^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$s_y^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$$

what is $\bar{y}$?

$$\bar{y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n} = \frac{\sum x_i - n\bar{x}}{n} = \bar{x} - \bar{x} = 0$$

so

$$s_y^2 = \frac{\sum_{i=1}^{n}(y_i - 0)^2}{n-1} = \frac{\sum_{i=1}^{n} y_i^2}{n-1}$$

standard deviation squared of y (ie variance)

$$= \frac{\sum (x_i - \bar{x})^2}{n-1} = s_x^2$$

standard deviation^2 of x (ie variance)

$$\Rightarrow s_y = s_x$$

(b) $z_i = (x_i - \bar{x})/s_x \quad i = 1, \ldots, n$

$$\bar{z} = \frac{\sum_{i=1}^{n} z_i}{n} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})/s_x}{n} = \frac{1}{n s_x}\left(\sum x_i - n\bar{x}\right) = 0$$

$$s_z^2 = \frac{\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-1} = \frac{\sum_{i=1}^{n} z_i^2}{n-1} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2 / s_x^2}{n-1}$$

$$= \frac{(n-1)s_x^2/s_x^2}{(n-1)} = 1 \quad \Rightarrow s_z = 1$$