

# Homework # 1 Selected Solutions

42(a) mean of the  $x_i$ 's  $\Rightarrow \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

median of the  $x_i$ 's  $\Rightarrow x_{[\frac{n}{2}]}$

This is notation for order statistics. It means the  $\frac{n}{2}$ th observation in an ordered list.

$$y_i = x_i + c$$

$$\bar{y} = \frac{\sum_{i=1}^n y_i}{n} = \frac{\sum_{i=1}^n (x_i + c)}{n} = \frac{\sum_{i=1}^n x_i + nc}{n} = \bar{x} + c$$

median of the  $y_i$ 's  $y_{[\frac{n}{2}]} = x_{[\frac{n}{2}]} + c$

(b)  $y_i = cx_i$

$$\bar{y} = \frac{\sum_{i=1}^n y_i}{n} = \frac{\sum_{i=1}^n cx_i}{n} = c \frac{\sum_{i=1}^n x_i}{n} = c\bar{x}$$

median of  $y_i$ 's  $y_{[\frac{n}{2}]} = cx_{[\frac{n}{2}]}$

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$\sum_{i=1}^n (x_i - c)^2$  want to find  $c$  which minimizes  
this value

$$\begin{aligned} \frac{d}{dc} \sum_{i=1}^n (x_i - c)^2 &= \sum_{i=1}^n \frac{d}{dc} (x_i - c)^2 \\ &= \sum_{i=1}^n 2(x_i - c)(-1) \end{aligned}$$

set = 0

$$-2 \sum_{i=1}^n (x_i - c) = 0$$

$$\Rightarrow \sum_{i=1}^n x_i - nc = 0$$

$$\Rightarrow \sum_{i=1}^n x_i = nc$$

$$\Rightarrow c = \frac{\sum_{i=1}^n x_i}{n} = \bar{x}$$

(b) by part (a)  $\sum_{i=1}^n (x_i - \bar{x})^2 \leq \sum_{i=1}^n (x_i - \mu)^2$

with equality only if  $\bar{x} = \mu$

$$7B(a) \quad y_i = x_i - \bar{x} \quad i = 1, \dots, n$$

Note  
 $s_x$   
 $s_y$  } standard deviations  
 $s_z$  } of  $x, y, z$

$$s_y^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}$$

$$s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

what is  $\bar{y}$ ?

$$\bar{y} = \frac{\sum_{i=1}^n (x_i - \bar{x})}{n} = \frac{\sum x_i - n\bar{x}}{n} = \bar{x} - \bar{x} = 0$$

$$s_y^2 = \frac{\sum_{i=1}^n (y_i - 0)^2}{n-1} = \frac{\sum_{i=1}^n y_i^2}{n-1}$$

Standard deviation squared of  $y$  (ie variance)

$$= \frac{\sum (x_i - \bar{x})^2}{n-1} = s_x^2 \leftarrow \text{standard deviation}^2 \text{ of } x \text{ (ie variance)}$$

$$\Rightarrow s_y = s_x$$

$$(b) \quad z_i = (x_i - \bar{x}) / s_x \quad i = 1, \dots, n$$

$$\bar{z} = \frac{\sum_{i=1}^n z_i}{n} = \frac{\sum_{i=1}^n (x_i - \bar{x}) / s_x}{n} = \frac{1}{ns_x} (\sum x_i - n\bar{x}) = 0$$

$$s_z^2 = \frac{\sum_{i=1}^n (z_i - \bar{z})^2}{n-1} = \frac{\sum_{i=1}^n z_i^2}{n-1} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2 / s_x^2}{n-1}$$

$$= \frac{(n-1) s_x^2 / s_x^2}{(n-1)} = 1 \Rightarrow s_z = 1$$