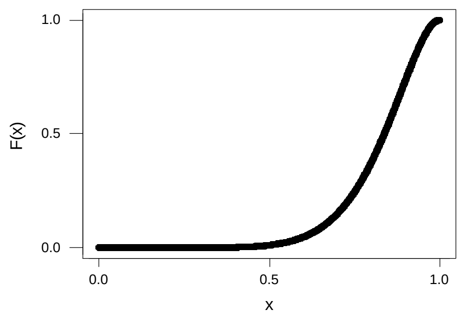
Homework #4 Solutions

Chapter 4 # 15

a. F(X) = 0 for x ≤ 0, = 1 for x ≥ 1, and for 0 < X < 1,

$$F(X) = \int_{-\infty}^{x} f(y)dy = \int_0^x 90y^8(1-y)dy = 90\int_0^x (y^8 - y^9)dy$$

$$90\left(\tfrac{1}{9}y^9 - \tfrac{1}{10}y^{10}\right)\Big]_0^x = 10x^9 - 9x^{10}$$



b. $F(.5) = 10(.5)^9 - 9(.5)^{10} \approx .0107$

c. $P(.25 \le X \le .5) = F(.5) - F(.25) \approx .0107 - [10(.25)^9 - 9(.25)^{10}]$
$\approx .0107 - .0000 \approx .0107$

d. The 75th percentile is the value of x for which F(x) = .75
$\Rightarrow .75 = 10(x)^9 - 9(x)^{10}$ $\Rightarrow x \approx .9036$

e. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^1 x \cdot 90x^8(1-x)dx = 90\int_0^1 x^9(1-x)dx$

$$= 9x^{10} - \tfrac{90}{11}x^{11}\Big]_0^1 = \tfrac{9}{11} \approx .8182$$

$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx = \int_0^1 x^2 \cdot 90x^8(1-x)dx = 90\int_0^1 x^{10}(1-x)dx$

$$= \tfrac{90}{11}x^{11} - \tfrac{90}{12}x^{12}\Big]_0^1 \approx .6818$$

$V(X) \approx .6818 - (.8182)^2 = .0124$, $\sigma_x = .11134$.

f. $\mu \pm \sigma = (.7068, .9295)$. Thus, $P(\mu - \sigma \le X \le \mu + \sigma) = F(.9295) - F(.7068)$
$= .8465 - .1602 = .6863$

Problem 22

a. For $1 \le x \le 2$, $F(x) = \int_1^x 2\left(1 - \frac{1}{y^2}\right)dy = 2\left(y + \frac{1}{y}\right)\Big]_1^x = 2\left(x + \frac{1}{x}\right) - 4$, so

$$F(x) = \begin{cases} 0 & x < 1 \\ 2\left(x + \frac{1}{x}\right) - 4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

b. $2\left(x_p + \frac{1}{x_p}\right) - 4 = p \Rightarrow 2x_p^2 - (4 - p)x_p + 2 = 0 \Rightarrow x_p = \frac{1}{4}[4 + p + \sqrt{p^2 + 8p}]$ To find $\tilde{m}$, set p = .5 ⇒ $\tilde{m}$ = 1.64

c. $E(X) = \int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right)dx = 2\int_1^2 \left(x - \frac{1}{x}\right)dx = 2\left(\frac{x^2}{2} - \ln(x)\right)\Big]_1^2 = 1.614$

$E(X^2) = 2\int_1^2 \left(x^2 - 1\right)dx = 2\left(\frac{x^3}{3} - x\right)\Big]_1^2 = \frac{8}{3} \Rightarrow$ Var(X) = .0626

d. Amount left = max(1.5 – X, 0), so

E(amount left) = $\int_1^2 \max(1.5 - x, 0)\, f(x)dx = 2\int_1^{1.5} (1.5 - x)\left(1 - \frac{1}{x^2}\right)dx = .061$

Problem 26

a. .9838 is found in the 2.1 row and the .04 column of the standard normal table so c = 2.14.

b. $P(0 \le Z \le c) = .291 \Rightarrow \Phi(c) = .7910 \Rightarrow c = .81$

c. $P(c \le Z) = .121 \Rightarrow 1 - P(c \le Z) = P(Z < c) = \Phi(c) = 1 - .121 = .8790 \Rightarrow c = 1.17$

d. $P(-c \le Z \le c) = \Phi(c) - \Phi(-c) = \Phi(c) - (1 - \Phi(c)) = 2\Phi(c) - 1$
$\Rightarrow \Phi(c) = .9920 \Rightarrow c = .97$

e. $P(c \le |Z|) = .016 \Rightarrow 1 - .016 = .9840 = 1 - P(c \le |Z|) = P(|Z| < c)$
$= P(-c < Z < c) = \Phi(c) - \Phi(-c) = 2\Phi(c) - 1$
$\Rightarrow \Phi(c) = .9920 \Rightarrow c = 2.41$

Problem 27

a. $\Phi(c) = .9100 \Rightarrow c \approx 1.34$ (.9099 is the entry in the 1.3 row, .04 column)

b. 9th percentile = -91st percentile = -1.34

c. $\Phi(c) = .7500 \Rightarrow c \approx .675$ since .7486 and .7517 are in the .67 and .68 entries, respectively.

d. 25th = -75th = -.675

e. $\Phi(c) = .06 \Rightarrow c \approx$.-1.555 (both .0594 and .0606 appear as the –1.56 and –1.55 entries, respectively).

Problem 33

Let X denote the diameter of a randomly selected cork made by the first machine, and let Y be defined analogously for the second machine.

$P(2.9 \le X \le 3.1) = P(-1.00 \le Z \le 1.00) = .6826$
$P(2.9 \le Y \le 3.1) = P(-7.00 \le Z \le 3.00) = .9987$

So the second machine wins handily.

Problem 51

$P(X \le \mu + \sigma[(100p)\text{th percentile for std normal}])$

$$P\left(\frac{X-m}{s} \le [\ldots]\right) = P(Z \le [\ldots]) = p \text{ as desired}$$

Problem 58

a. $E(X) = \frac{1}{l} = 1$

b. $s = \frac{1}{l} = 1$

c. $P(X \le 4) = 1 - e^{-(1)(4)} = 1 - e^{-4} = .982$

d. $P(2 \le X \le 5) = 1 - e^{-(1)(5)} - \left[1 - e^{-(1)(2)}\right] = e^{-2} - e^{-5} = .129$

Problem 59

a. $P(X \le 100) = 1 - e^{-(100)(.01386)} = 1 - e^{-1.386} = .7499$
$P(X \le 200) = 1 - e^{-(200)(.01386)} = 1 - e^{-2.772} = .9375$
$P(100 \le X \le 200) = P(X \le 200) - P(X \le 100) = .9375 - .7499 = .1876$

b. $\mu = \frac{1}{.01386} = 72.15$, $\sigma = 72.15$
$P(X > \mu + 2\sigma) = P(X > 72.15 + 2(72.15)) = P(X > 216.45) =$
$1 - \left[1 - e^{-(216.45)(.01386)}\right] = e^{-2.9999} = .0498$

c. $.5 = P(X \le \tilde{m}) \Rightarrow 1 - e^{-(\tilde{m})(.01386)} = .5 \Rightarrow e^{-(\tilde{m})(.01386)} = .5$
$-\tilde{m}(.01386) = \ln(.5) = .693 \Rightarrow \tilde{m} = 50$

Chapter 5 Problem 1

a. $P(X = 1, Y = 1) = p(1,1) = .20$

b. $P(X \le 1 \text{ and } Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .42$

c. At least one hose is in use at both islands. $P(X \ne 0 \text{ and } Y \ne 0) = p(1,1) + p(1,2) + p(2,1) + p(2,2) = .70$

d. By summing row probabilities, $p_x(x) = .16, .34, .50$ for $x = 0, 1, 2$, and by summing column probabilities, $p_y(y) = .24, .38, .38$ for $y = 0, 1, 2$. $P(X \le 1) = p_x(0) + p_x(1) = .50$

e. $P(0,0) = .10$, but $p_x(0) \cdot p_y(0) = (.16)(.24) = .0384 \ne .10$, so X and Y are not independent.

Problem 2

a.

	p(x,y)	y = 0	1	2	3	4	
x	0	.30	.05	.025	.025	.10	.5
	1	.18	.03	.015	.015	.06	.3
	2	.12	.02	.01	.01	.04	.2
		.6	.1	.05	.05	.2	

b. $P(X \le 1 \text{ and } Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .56$
$= (.8)(.7) = P(X \le 1) \cdot P(Y \le 1)$

c. $P(X + Y = 0) = P(X = 0 \text{ and } Y = 0) = p(0,0) = .30$

d. $P(X + Y \le 1) = p(0,0) + p(0,1) + p(1,0) = .53$