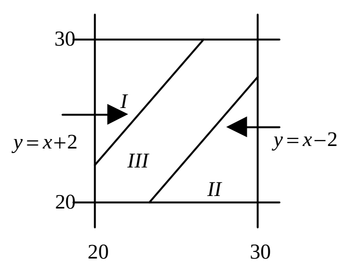
Homework #5 Solutions

Chapter 5 Problem 9

a. $$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dxdy = \int_{20}^{30}\int_{20}^{30} K(x^2+y^2)\,dxdy$$
$$= K\int_{20}^{30}\int_{20}^{30} x^2\,dydx + K\int_{20}^{30}\int_{20}^{30} y^2\,dxdy = 10K\int_{20}^{30} x^2\,dx + 10K\int_{20}^{30} y^2\,dy$$
$$= 20K\cdot\left(\frac{19{,}000}{3}\right) \Rightarrow K = \frac{3}{380{,}000}$$

b. P(X < 26 and Y < 26) = $\int_{20}^{26}\int_{20}^{26} K(x^2+y^2)\,dxdy = 12K\int_{20}^{26} x^2\,dx$
$$4Kx^3\Big|_{20}^{26} = 38{,}304K = .3024$$

c.



P(| X – Y | ≤ 2) = $\iint_{\substack{region\\III}} f(x,y)\,dxdy$
$$1 - \iint_{I} f(x,y)\,dxdy - \iint_{II} f(x,y)\,dxdy$$
$$1 - \int_{20}^{28}\int_{x+2}^{30} f(x,y)\,dydx - \int_{22}^{30}\int_{20}^{x-2} f(x,y)\,dydx$$
= (after much algebra) .3593

d. $f_x(x)$ = $\int_{-\infty}^{\infty} f(x,y)\,dy = \int_{20}^{30} K(x^2+y^2)\,dy = 10Kx^2 + K\frac{y^3}{3}\Big|_{20}^{30}$

= $10Kx^2 + .05$, $20 \le x \le 30$

e. $f_y(y)$ is obtained by substituting y for x in (d); clearly $f(x,y) \ne f_x(x)\cdot f_y(y)$, so X and Y are not independent.

Problem 19

f. $f_{Y|X}(y \mid x) = \dfrac{f(x,y)}{f_X(x)} = \dfrac{k(x^2+y^2)}{10kx^2+.05}$ $\quad 20 \le y \le 30$

$f_{X|Y}(x \mid y) = \dfrac{k(x^2+y^2)}{10ky^2+.05}$ $\quad 20 \le x \le 30 \quad \left(k = \dfrac{3}{380{,}000}\right)$

g. P(Y ≥ 25 | X = 22) = $\int_{25}^{30} f_{Y|X}(y \mid 22)\,dy$

$= \int_{25}^{30} \dfrac{k((22)^2+y^2)}{10k(22)^2+.05}\,dy = .783$

P(Y ≥ 25) = $\int_{25}^{30} f_Y(y)\,dy = \int_{25}^{30} (10ky^2+.05)\,dy = .75$

h. E(Y | X=22) = $\int_{-\infty}^{\infty} y \cdot f_{Y|X}(y \mid 22)\,dy = \int_{20}^{30} y \cdot \dfrac{k((22)^2+y^2)}{10k(22)^2+.05}\,dy$

= 25.372912

E(Y^2 | X=22) = $\int_{20}^{30} y^2 \cdot \dfrac{k((22)^2+y^2)}{10k(22)^2+.05}\,dy = 652.028640$

V(Y| X = 22) = E(Y^2 | X=22) – [E(Y | X=22)]^2 = 8.243976

Problem 22

a. E(X + Y) = $\sum_x \sum_y (x+y)\,p(x,y) = (0+0)(.02)$

$+ (0+5)(.06) + \ldots + (10+15)(.01) = 14.10$

b. E[max (X,Y)] = $\sum_x \sum_y \max(x+y) \cdot p(x,y)$

$= (0)(.02) + (5)(.06) + \ldots + (15)(.01) = 9.60$

Problem 30

a. E(X) = 5.55, E(Y) = 8.55, E(XY) = (0)(.02) + (0)(.06) + … + (150)(.01) = 44.25, so Cov(X,Y) = 44.25 – (5.55)(8.55) = -3.20

b. $s_X^2 = 12.45, s_Y^2 = 19.15$, so $r_{X,Y} = \dfrac{-3.20}{\sqrt{(12.45)(19.15)}} = -.207$

Problem 37

	$P(x_1)$	.20	.50	.30
$P(x_2)$	$x_2 \mid x_1$	25	40	65
.20	25	.04	.10	.06
.50	40	.10	.25	.15
.30	65	.06	.15	.09

a.

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	.04	.20	.25	.12	.30	.09

$$E(\bar{x}) = (25)(.04) + 32.5(.20) + \ldots + 65(.09) = 44.5 = m$$

b.

s^2	0	112.5	312.5	800
$P(s^2)$	.38	.20	.30	.12

$E(s^2) = 212.25 = \sigma^2$

Problem 49

a. 11 P.M. – 6:50 P.M. = 250 minutes. With $T_0 = X_1 + \ldots + X_{40}$ = total grading time, $m_{T_0} = nm = (40)(6) = 240$ and $s_{T_0} = s\sqrt{n} = 37.95,$ so $P(T_0 \le 250) \approx$

$$P\left(Z \le \frac{250-240}{37.95}\right) = P(Z \le .26) = .6026$$

b. $$P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > .53) = .2981$$

Problem 59

c. $E(X_1 + X_2 + X_3) = 180$, $V(X_1 + X_2 + X_3) = 45$, $s_{x_1+x_2+x_3} = 6.708$

$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{200-180}{6.708}\right) = P(Z \le 2.98) = .9986$

$P(150 \le X_1 + X_2 + X_3 \le 200) = P(-4.47 \le Z \le 2.98) \approx .9986$

d. $m_{\overline{X}} = m = 60$, $s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{\sqrt{15}}{\sqrt{3}} = 2.236$

$$P(\overline{X} \geq 55) = P\left(Z \geq \frac{55-60}{2.236}\right) = P(Z \geq -2.236) = .9875$$

$$P(58 \leq \overline{X} \leq 62) = P(-.89 \leq Z \leq .89) = .6266$$

e. $E(X_1 - .5X_2 - .5X_3) = 0$;

$V(X_1 - .5X_2 - .5X_3) = s_1^2 + .25s_2^2 + .25s_3^2 = 22.5$, sd = 4.7434

$P(-10 \leq X_1 - .5X_2 - .5X_3 \leq 5) = P\left(\frac{-10-0}{4.7434} \leq Z \leq \frac{5-0}{4.7434}\right)$

$= P(-2.11 \leq Z \leq 1.05) = .8531 - .0174 = .8357$

f. $E(X_1 + X_2 + X_3) = 150$, $V(X_1 + X_2 + X_3) = 36$, $s_{x_1+x_2+x_3} = 6$

$P(X_1 + X_2 + X_3 \leq 200) = P\left(Z \leq \frac{160-150}{6}\right) = P(Z \leq 1.67) = .9525$

We want $P(X_1 + X_2 \geq 2X_3)$, or written another way, $P(X_1 + X_2 - 2X_3 \geq 0)$

$E(X_1 + X_2 - 2X_3) = 40 + 50 - 2(60) = -30$,

$V(X_1 + X_2 - 2X_3) = s_1^2 + s_2^2 + 4s_3^2 = 78$, 36, sd = 8.832, so

$P(X_1 + X_2 - 2X_3 \geq 0) = P\left(Z \geq \frac{0-(-30)}{8.832}\right) = P(Z \geq 3.40) = .0003$

Problem 71

a. $M = a_1X_1 + a_2X_2 + W\int_0^{12} x\,dx = a_1X_1 + a_2X_2 + 72W$, so

$E(M) = (5)(2) + (10)(4) + (72)(1.5) = 158\text{m}$

$s_M^2 = (5)^2(.5)^2 + (10)^2(1)^2 + (72)^2(.25)^2 = 430.25$, $s_M = 20.74$

b. $P(M \leq 200) = P\left(Z \leq \frac{200-158}{20.74}\right) = P(Z \leq 2.03) = .9788$

Problem 84

I have 192 oz. The amount which I would consume if there were no limit is $T_o = X_1 + \ldots + X_{14}$ where each X_I is normally distributed with $\mu = 13$ and $\sigma = 2$. Thus T_o is normal with $m_{T_o} = 182$ and $s_{T_o} = 7.483$, so $P(T_o < 192) = P(Z < 1.34) = .9099$.

Chapter 6 Problem 1

a. We use the sample mean, $\overline{x}$ to estimate the population mean m .

$$\hat{m} = \overline{x} = \frac{\Sigma x_i}{n} = \frac{219.80}{27} = 8.1407$$

b. We use the sample median, $\tilde{x} = 7.7$ (the middle observation when arranged in ascending order).

c. We use the sample standard deviation, $s = \sqrt{s^2} = \sqrt{\frac{1860.94 - \frac{(219.8)^2}{27}}{26}} = 1.660$

d. With "success" = observation greater than 10, x = # of successes = 4, and $\hat{p} = \frac{x}{n} = \frac{4}{27} = .1481$

e. We use the sample (std dev)/(mean), or $\frac{s}{\overline{x}} = \frac{1.660}{8.1407} = .2039$