Homework #7 Solutions

Chapter 8 Problem 4

When the alternative is H_a: $m < 5$, the formulation is such that the water is believed unsafe until proved otherwise. A type I error involved deciding that the water is safe (rejecting H_o) when it isn't (H_o is true). This is a very serious error, so a test which ensures that this error is highly unlikely is desirable. A type II error involves judging the water unsafe when it is actually safe. Though a serious error, this is less so than the type I error. It is generally desirable to formulate so that the type 1 error is more serious, so that the probability of this error can be explicitly controlled. Using H_a: $m > 5$, the type II error (now stating that the water is safe when it isn't) is the more serious of the two errors.

Problem 8

Let $m_1 =$ the average amount of warpage for the regular laminate, and $m_2 =$ the analogous value for the special laminate. Then the hypotheses are $H_o : m_1 = m_2$ vs $H_o : m_1 > m_2$. Type I error: Conclude that the special laminate produces less warpage than the regular, when it really does not. Type II error: Conclude that there is no difference in the two laminates when in reality, the special one produces less warpage.

Problem 21

With H_o: $m = .5$, and H_a: $m \neq .5$ we reject H_o if $t > t_{a/2,n-1}$ or $t < -t_{a/2,n-1}$

a. 1.6 < $t_{.025,12}$ = 2.179, so don't reject H_o

b. -1.6 > -$t_{.025,12}$ = -2.179, so don't reject H_o

c. – 2.6 > -$t_{.005,24}$ = -2.797, so don't reject H_o

d. –3.9 < the negative of all t values in the df = 24 row, so we reject H_o in favor of H_a.

Problem 30

n = 115, $\bar{x} = 11.3$, $s = 6.43$

1 Parameter of Interest: $m =$ true average dietary intake of zinc among males aged 65 – 74 years.

2 Null Hypothesis: H_o: $m = 15$

3 Alternative Hypothesis: H_a: $m < 15$

4 $$z = \frac{\bar{x} - m_o}{s/\sqrt{n}} = \frac{\bar{x} - 15}{s/\sqrt{n}}$$

5 Rejection Region: No value of α was given, so select a reasonable level of significance, such as α= .05. $z \leq z_a$ or $z \leq -1.645$

6 $z = \frac{11.3 - m_o}{6.43/\sqrt{115}} = -6.17$

7 –6.17 < -1.645, so reject H_o. The data does support the claim that average daily intake of zinc for males aged 65 - 74 years falls below the recommended daily allowance of 15 mg/day.

Problem 31

The hypotheses of interest are H_o: $m = 7$ vs H_a: $m < 7$, so a lower-tailed test is appropriate; H_o should be rejected if $t \le -t_{.1,8} = -1.397$. $t = \frac{6.32 - 7}{1.65/\sqrt{9}} = -1.24$. Because -1.24 is not ≤ -1.397, H_o (prior belief) is not rejected (contradicted) at level .01.

Problem 53

p = proportion of all physicians that know the generic name for methadone.
H_o: p = .50 vs H_a: p < .50; We can use a large sample test if both $np_0 \ge 10$ and $n(1 - p_0) \ge 10$; 102(.50) = .51, so we can proceed. $\hat{p} = \frac{47}{102}$, so $z = \frac{\frac{47}{102} - .50}{\sqrt{\frac{(.50)(.50)}{102}}} = \frac{-.039}{.050} = -.79$. We will reject H_0 if the p-value < .01. For this lower tailed test, the p-value = Φ(z) = Φ(-.79) =.2148, which is not < .01, so we do not reject H_0 at significance level .01.

Chapter 9 Problem 8

1 Parameter of interest: $m_1 - m_2 =$ the true difference of mean tensile strength of the 1064 grade and the 1078 grade wire rod. Let $m_1 =$ 1064 grade average and $m_2 =$ 1078 grade average.

2 H_o: $m_1 - m_2 = -10$

3 H_a: $m_1 - m_2 < -10$

4 $z = \frac{(\bar{x} - \bar{y}) - \Delta_o}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}} = \frac{(\bar{x} - \bar{y}) - (-10)}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$

5 RR: $p - value < a$

6 $z = \frac{(107.6 - 123.6) - (-10)}{\sqrt{\frac{1.3^2}{129} + \frac{2.0^2}{129}}} = \frac{-6}{.210} = -28.57$

7 For a lower-tailed test, the p-value = $\Phi(-28.57) \approx 0$, which is less than any a , so reject H_o. There is very compelling evidence that the mean tensile strength of the 1078 grade exceeds that of the 1064 grade by more than 10.

b. The requested information can be provided by a 95% confidence interval for $m_1 - m_2$:

$$(\bar{x} - \bar{y}) \pm 1.96\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}} = (-6) \pm 1.96(.210) = (-6.412, -5.588)$$.

Problem 29

Let $m_1 =$ the true average compression strength for strawberry drink and let $m_2 =$ the true average compression strength for cola. A lower tailed test is appropriate. We test $H_0 : m_1 - m_2 = 0$ vs. $H_a : m_1 - m_2 < 0$. The test statistic is $t = \frac{-14}{\sqrt{29.4 + 15}} = -2.10$.

$$n = \frac{(44.4)^2}{\frac{(29.4)^2}{14} + \frac{(15)^2}{14}} = \frac{1971.36}{77.8114} = 25.3$$, so use df=25. The p-value $\approx P(t < -2.10) = .023$. This p-value indicates strong support for the alternative hypothesis. The data does suggest that the extra carbonation of cola results in a higher average compression strength.

Problem 40

a. H_o will be rejected in favor of H_a if either $t \geq t_{.005,15} = 2.947$ or $t \leq -2.947$. The summary quantities are $\bar{d} = -.544$, and $s_d = .714$, so $t = \frac{-.544}{.1785} = -3.05$. Because $-3.05 \leq -2.947$, H_o is rejected in favor of H_a.

b. $s_p^2 = 7.31$, $s_p = 2.70$, and $t = \frac{-.544}{.96} = -.57$, which is clearly insignificant; the incorrect analysis yields an inappropriate conclusion.

Problem 49

1 Parameter of interest: $p_1 - p_2$ = true difference in proportions of those responding to two different survey covers. Let p_1 = Plain, p_2 = Picture.

2 $H_0 : p_1 - p_2 = 0$

3 $H_a : p_1 - p_2 < 0$

4 $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}}$

5 Reject H_o if p-value < .10

6 $$z = \frac{\frac{104}{207} - \frac{109}{213}}{\sqrt{\left(\frac{213}{420}\right)\left(\frac{207}{420}\right)\left(\frac{1}{207} + \frac{1}{213}\right)}} = -.1910$$; p-value = .4247

7 Fail to Reject H_o. The data does not indicate that plain cover surveys have a lower response rate.